

Roll No.

D-989**M. A./M. Sc. (Fourth Semester) (Main/ATKT)****EXAMINATION, May-June, 2020**

MATHEMATICS

(Optional—A)

Paper Fourth

(Operations Research—II)*Time : Three Hours] [Maximum Marks : 80***Note :** Attempt all Sections as directed.**Section—A** 1 each**(Objective/Multiple Choice Questions)****Note :** Attempt all questions.

Choose the correct answer :

1. Dynamic programming deals with the :

- (a) Multi-stage decision-making problems.
- (b) Single stage decision-making problems.
- (c) Time independence decision-making problems.
- (d) None of these

2. When a quantity k is divided into n parts, the maximum value of their product is :

- (a) $\left(\frac{k}{n}\right)^n$
- (b) $n\left(\frac{k}{n}\right)$
- (c) $(nk)^n$
- (d) $n(nk)$

3. Identify the wrong sentence :

- (a) Dynamic programming can also deal with non-linear constraints.
- (b) Dynamic programming can be solved by simplex method.
- (c) Dynamic programming can be divided into a sequence of smaller subproblem.
- (d) None of these

4. The solution of a dynamic programming problem is largely base on the principle of optimality.

- (a) True
- (b) False

5. A two person game is said to be zero-sum if :

- (a) the algebraic sum of gains and losses of all the players is zero.
- (b) the algebraic sum of gains and losses of all the players is not zero.
- (c) the elements in the diagonal of the pay of matrix is zero.
- (d) Each player has a finite number of strategies to him.

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6. Choose the wrong statement :
- (a) The game has a saddle point if the pay-off associated with maximin and minimax strategies are same.
 - (b) The game value can be positive, negative or zero.
 - (c) The games is strictly determinable and fair if saddle point comes out to be zero.
 - (d) The value of game can be determined only if the game has a saddle point.
7. For a two-person game, where A and B respectively, are the minimising and the maximising players, the optimum strategies are :
- (a) Minimax for A and maximin for B
 - (b) Minimin for A and maximax for B
 - (c) Maximin for A and minimax for B
 - (d) None of these
8. Choose the correct statement :
- (a) A game does not have more than one saddle point.
 - (b) Dominance property is applied when there is no saddle point.
 - (c) A two person game cannot be converted into a linear programming problem.
 - (d) If the strategies are mixed, the value of game for both players need not be the same.

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9. The value of the game whose payoff matrix is $\begin{bmatrix} 3 & 11 \\ 5 & 2 \end{bmatrix}$ is :
- (a) $\frac{39}{11}$
 - (b) $\frac{49}{11}$
 - (c) $\frac{59}{11}$
 - (d) 0
10. In Gomory's all-integer cutting plane method the objective function reformulate in standard :
- (a) Minimization form
 - (b) Maximization form
 - (c) Minimization or maximization form
 - (d) None of these
11. Consider the following problem :
- Maximize :
- $$z = 5x + 8y$$
- subject to the constraints :
- $$3x + 2y \geq 3$$
- $$x + y \leq 5$$
- $$x + 4y \geq 4$$
- and $x \geq 0, y \geq 0$.
- This problem is :
- (a) a 0 – 1 IPP
 - (b) a pure IPP
 - (c) a mixed IPP
 - (d) not in IPP

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12. In cutting plane algorithm, each cut which is made involves the introduction of :

- (a) a less than equal to constraint.
- (b) a greater than equal to constraint.
- (c) an equality constraints.
- (d) an artificial variable.

13. Identify the statement which is not correct in the context of Branch and Bound method :

- (a) It divides the feasible region into smaller parts by the process of branching.
- (b) It can be used to solve any kind of programming problem.
- (c) It is a not a particular method and is used differently in different kinds of problems.
- (d) It terminates when the upper and lower bounds become identical.

14. The following problem :

Maximize :

$$z = x_1 + 4x_2$$

subject to :

$$2x_1 + 4x_2 \leq 7$$

$$5x_1 + 3x_2 \leq 15$$

$$x_1, x_2 \geq 0$$

and are integers.

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This problem is :

- (a) a pure IPP
- (b) a mixed IPP
- (c) not in IPP
- (d) a 0 – 1 IPP

15. For a Poisson exponential-single server infinite population mode, which of the following is not true ?

- (a) The arrivals are according to Poisson distributed.
- (b) The service rate has exponential-distributed.
- (c) The source population is a small sized finite.
- (d) The system has a single service facility.

16. Identify the correct answer :

- (a) A flow of customers towards the service facility form a queue on account of lack of capability to serve them all at a time.
- (b) Queue is formed only when the current demand for the service exceeds the capacity of the service facility to render service.
- (c) A queue refers to the physical presence of the customers waiting to be served.
- (d) In a queuing situation, the customers are always human being.

17. In banking transaction, tellers represent the service station.

- (a) True
- (b) False

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18. Optimum solution to a NLPP using Kuhn-Tucker condition and graphical method are not same.
- (a) True
(b) False
19. Quadratic programming is concerned with the NLPP of optimizing the quadratic objective function subject to :
- (a) non-linear equality constraints
(b) non-linear inequality constraints
(c) linear inequality constraints
(d) no constraints
20. Which of the following methods of solving quadratic programming problem is based on modified simplex method ?
- (a) Beale's method
(b) Wolfe's method
(c) Fletcher's method
(d) None of these

Section—B

2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Define dynamic programming.
2. What is two-person zero-sum game ?
3. State the rules for determining a saddle point.

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4. Determine the minimax and maximin value for the following matrix :

$$\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 3 \\ 6 & 2 & 1 \end{bmatrix}$$

5. Give an example of mixed integer programming.
6. What do you understand by Queuing Problems ?
7. Define non-linear programming problem.
8. What do you understand by Quadratic Programming ?

Section—C

3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. Write the dynamic programming algorithm.
2. Find the optimum strategies for players A and B :

B

$$A \begin{bmatrix} 0 & 2 \\ -1 & 4 \end{bmatrix}$$

3. Solve the game with the following payoff matrix. Determine the optimal strategies and value of game :

B

$$A \begin{bmatrix} 6 & -3 \\ -3 & 0 \end{bmatrix}$$

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4. Does the following game have a saddle point ?

$$\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$$

5. Find the optimum solution to the LPP :

Maximize :

$$z = x_1 - 2x_2$$

subject to the constraints :

$$4x_1 + 2x_2 \leq 15$$

$$x_1, x_2 \geq 0$$

and are integers.

6. In a super market the average arrival rate of customers is 5 every 30 minutes. The average time it takes to list and calculate the customer's purchases at the cash desk is 4.5 minutes and this time is exponential distributed. What is the chance that the queue length will exceed 5 ?
7. A drive-in bank window has a mean service time of 2 minutes, while the customers arrive at a rate of 20 per hour. Assuming that these represent rate with a Poisson distribution, determine the proportion the teller will be idle.
8. Obtain necessary condition for the optimum solution of the following problem :

Minimize :

$$f(x_1, x_2) = 3 \exp(2x_1 + 1) + 2 \exp(x_2 + 5)$$

subject to the constraints :

$$g(x_1, x_2) = x_1 + x_2 - 7 = 0.$$

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Section—D

4 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. Use dynamic programming to find the value of :

Maximize :

$$z = y_1 \cdot y_2 \cdot y_3$$

subject to the constraints :

$$y_1 + y_2 + y_3 = 5$$

$$y_1, y_2, y_3 \geq 0.$$

Or

Minimize :

$$z = y_1^2 + y_2^2 + y_3^2$$

subject to the constraints :

$$y_1 + y_2 + y_3 \geq 15$$

$$y_1, y_2, y_3 \geq 0.$$

2. Solve the following 2×3 game graphically :

Player B

$$\text{Player A} \begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$$

Or

Use dominance property to solve the following game :

Player B

$$\text{Player A} \begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{bmatrix}$$

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3. Find the integer solution to the L. P. P. :

Maximize :

$$z = 2x_1 + 2x_2$$

subject to the constraints :

$$5x_1 + 3x_2 \leq 8$$

$$x_1 + 2x_2 \leq 4$$

$x_1, x_2 \geq 0$ and are integers.

Or

Solve the following mixed integer programming problem :

Maximize :

$$z = x_1 + x_2$$

subject to the constraints :

$$3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

x_1 is an integer.

4. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes, calculate the following :
- the mean queue size (length)
 - the probability that the size exceeds 10.

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Or

A super market has two girls running up sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in a Poisson fashion at the rate of 10 an hour, what is the probability of having to wait for service ?

5. Use the Kuhn-Tucker condition to solve the following non-linear programming problem :

Maximize :

$$z = 2x_1^2 + 12x_1x_2 - 7x_2^2$$

subject to the constraints :

$$2x_1 + 5x_2 \leq 98$$

$$x_1, x_2 \geq 0.$$

Or

Use Wolfe's method to solve the following Q. P. P. :

Maximize :

$$z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

subject to the constraints :

$$x_1 + 2x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

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